Chapter 3 Suggestions on Learning and Teaching

Guiding Principles

In designing learning and teaching activities in Additional Mathematics, the following principles should be noted:

- Our main concern is to help students *learn to learn* rather than merely delivering subject contents to students.
- All students can learn although with different paces.
- A learner-focused approach should be adopted.
- Information technology, when used appropriately, would increase the effectiveness of learning and teaching.

Suggested Sequence

There are 9 units in the curriculum and they are presented in the following sequence in this curriculum guide:

Units

1. Principle of Mathematical Induction
2. Binomial Theorem for Positive Integral Indices
3. Quadratic Equations and Quadratic Functions
4. Trigonometry
5. Limits and Differentiation
6. Inequalities
7. Coordinate Geometry
8. Vectors in the Two-dimensional Space
9. Integration

Teachers should note that the sequence presented here only serves as an example. They are free to design their own teaching sequences to suit the needs of their students. When designing a school-based curriculum of the subject, teachers should ensure that students have already possessed the pre-requisite knowledge of the relevant topics before learning the topics concerned.

In order to achieve the learning objectives and realize the spirit of the curriculum, teachers are advised to teach the curriculum as a connected body of mathematical knowledge as far as possible and encourage students to inquire, reason and communicate mathematically.

Suggested Time Allocation

The suggested time allocation for the course is ***4 periods per week***. It is assumed that there are 40 minutes in each period and 5 days in a week. The suggested time ratio is given here to aid teachers in judging how far to take for a given topic. Schools may choose an equivalent or slightly different time allocation to suit their own situations.

	Units	**Time Ratio**	
1.	Principle of Mathematical Induction		10
2.	Binomial Theorem for Positive Integral Indices		11
3.	Quadratic Equations and Quadratic Functions	9*+	12
4.	Trigonometry	18*+	28
5.	Limits and Differentiation		44
6.	Inequalities	3*+	4
7.	Coordinate Geometry	12*+	35
8.	Vectors in the Two-dimensional Space		20
9.	Integration		22
		42*+	186

Note: The elementary parts of Units 3, 4, 6 and 7, which can also be found in the Secondary Mathematics Syllabus, are marked with asterisks (*). These parts form pre-requisite for more advanced parts of Units 3, 4, 6 and 7.

The time ratio given in each unit is the numerator of a fraction whose denominator, 200, is related to the total time spent on teaching the subject during the two years. The total time of "200" excludes the time spent on class tests and school examinations. The remaining time of "14" may be used for consolidation activities, enrichment activities or revision at the end of the course. It is intended that the time ratio will indicate the fraction of the total time to be spent on the units and sub-units concerned.

The following show the detailed breakdown of the units and the corresponding time ratios:

		Time Ratio
Unit 1	**Principle of Mathematical Induction**	
1.1	Concept of Mathematical Induction	2
1.2	Steps in the Method of Mathematical Induction	3
1.3	Application of Mathematical Induction	5
		10
Unit 2	**Binomial Theorem for Positive Integral Indices**	
2.1	The n! and C_r^n Notations	1
2.2	The Pascal Triangle	1
2.3	Expanding Binomials Using the Pascal Triangle	2
2.4	Binomial Theorem for Positive Integral Indices	7
		11
Unit 3	**Quadratic Equations and Quadratic Functions**	
3.1	Solution of Quadratic Equations	9*
3.2	Nature of Roots	5
3.3	Quadratic Functions	5
3.4	Absolute Values	2
		9* + 12
Unit 4	**Trigonometry**	
4.1	Radian Measure	3*
4.2	The Six Trigonometric Functions of Angles of Any Magnitude and their Graphs	5* + 5
4.3	Compound Angles	9
4.4	The Subsidiary Angle Form	3
4.5	General Solution of Simple Trigonometric Equations	5
4.6	Solution of Triangles	10*
4.7	Problems in Two and Three Dimensions	6
		18*+ 28

		Time Ratio
Unit 5	**Limits and Differentiation**	
5.1	Limits	6
5.2	Derivatives	5
5.3	Differentiation	
5.3.1	Simple Algebraic Functions and Rules of Differentiation	5
5.3.2	Differentiation of Composite Functions and Implicit Functions	6
5.3.3	Differentiation of Trigonometric Functions	5
5.3.4	Second Derivatives	2
5.4	Applications of Differentiation	
5.4.1	Gradients, Tangents and Normals to a Curve	5
5.4.2	Maxima and Minima, Sketching of Simple Curves	7
5.4.3	Rates of Change	3
		44

Unit 6	**Inequalities**	
6.1	Basic Rules of Inequalities	1*
6.2	Linear Inequalities in One Variable	1* + 1
6.3	Quadratic Inequalities in One Variable	1* + 3
		3* + 4

Unit 7	**Coordinate Geometry**	
7.1	Plane Rectangular Coordinates and Distance between Two Points	1*
7.2	Area of Rectilinear Figures	3
7.3	Inclinations and Slopes of Straight Lines	1*
7.4	Angle between Two Straight Lines	4
7.5	Equations of Straight Lines	1*
7.6	The Normal Form	5
7.7	Family of Straight Lines	5
7.8	Concept of Locus	1* + 3

		Time Ratio
7.9	Equation of a Circle, Intersection of a Straight Line and a Circle	8*
7.10	Equation of Tangents to a Circle	5
7.11	Family of Circles	5
7.12	Simple Parametric Equations and Locus Problems	5
		12* +35

Unit 8 Vectors in the Two-dimensional Space

8.1	Scalar and Vector Quantities, Equality of Vectors, Zero Vector and Unit Vector	3
8.2	Sum and Difference of Vectors, Multiplication of a Vector by a Scalar	4
8.3	Representation of Vectors in the Rectangular Coordinate System	3
8.4	Scalar Product of Two Vectors	5
8.5	Applications of Vectors, Division of a Line Segment, Parallelism and Perpendicularity	5
		20

Unit 9 Integration

9.1	Indefinite Integral	2
9.2	Integration of Simple Functions and Simple Applications	5
9.3	Simple Techniques of Integration	3
9.4	Definite Integral	2
9.5	Evaluation of Definite Integrals	3
9.6	Applications of Definite Integrals	7
		22

Total: 42*+186

Note: Teachers teaching Additional Mathematics but not teaching Secondary Mathematics to their classes should ensure that these pre-requisite sub-units are taught or ample revision given before starting the more advanced parts of these units. This arrangement of topics in relation to the secondary mathematics curriculum renders it an advantage for the same teacher to teach both Secondary Mathematics and Additional Mathematics to a class,

although schools may find it sometimes administratively more convenient to have two separate teachers teaching the two subjects in the same class.

Suggested Learning and Teaching Strategies

Both the learning process and the end product in the learning of Additional Mathematics are important. Students should be allocated sufficient time to develop mathematical concepts, master problem solving skills and foster thinking abilities. It should be noted that Additional Mathematics is no difference from the Secondary Mathematics *per se*. It only consists of different mathematical contents or contents of Secondary Mathematics with deeper treatment. Therefore, the learning and teaching strategies equally hold in both the curricula of Secondary Mathematics and Additional Mathematics.

It should be noted that no matter what emphasis of strategies is adopted, the teacher is the key person in classroom teaching. Liveliness and clear explanation of the teachers are students' main concern. Diversified learning and teaching activities are definitely beneficial to students. Applications of mathematical concepts to real-life situations also provide students motivation for learning Additional Mathematics. To help students develop the capability of appreciating the cultural aspect of mathematics, it is worthwhile to introduce related achievements of some mathematicians like Pascal (Pascal's Triangle or 楊輝三角 in Chinese), Descartes (coordinate geometry), Leibniz and Newton (calculus), etc.

In particular, attention should be paid to the following strategies when designing and preparing learning and teaching activities to facilitate students' learning.

<u>Catering for Learner Differences</u>

The curriculum is designed for more able students who are preparing to study Pure Mathematics or science stream in the sixth form. Basically, the difference in abilities of students studying this course should be less significant. However, under the policy of the nine-year free education, more students can study this subject resulting in the widening of the learner differences.

There is no hard and fast rule to address the problem of learner differences. However, the general approach of providing students with tasks or activities of different levels of difficulties seems viable. For less able students, tasks should be relatively simple and fundamental in nature. For more able ones, tasks assigned should be challenging enough to cultivate and sustain their interest in learning. Alternatively, teachers could provide students

with the same task, but vary the amount and style of support they give, i.e. giving more clues, breaking the more complicated problems into small parts for weaker students.

The use of information technology (IT) could also provide another solution for teachers to cater for learner differences. For some topics, such as the graphs of trigonometric functions, extremum points of curves, the use of appropriate software packages provide a simple, fast and accurate presentation which traditional teaching cannot fulfill. The use of IT in the learning of the subject is especially important to weaker students.

Appropriate Use of Information Technology

For many years, teaching additional mathematics is always conducted with chalk and talk. Until recently, the wide-spread use of computers provides enhancement for the learning and teaching in additional mathematics. Using IT in learning and teaching mathematics may bring about the following benefits:

(a) IT can enhance and extend mathematics learning experience, and encourage active student participation in exploratory and investigative activities.

(b) IT, when used as a tool, can support, supplement and extend learning and teaching activities, such as:
- exercises and tutorials;
- charting and graphical analysis;
- simulation and modeling;
- information retrieval and handling; and
- data processing.

(c) IT may lead to new teaching strategies and practices in classrooms such as providing students with an interactive learning environment for contextual and situational learning.

IT in mathematics education could be considered as:

(i) a tool - Teachers could use presentation software to present notes, geometry software to demonstrate graphs and models, zoom-in and zoom-out facilities in some graphing calculators or graph plotter software to sketch the graphs of different functions. For example, teachers could use computer program to demonstrate some complicated formulae in trigonometry. Teachers could also use *Java Applets* provided on the web to present interactive proofs.

(ii) a tutor - Many mathematical software packages, in the form of CD-ROMs, act as a tutor to teach students mathematical concepts. These software packages illustrate mathematical concepts with texts, graphics and sound and contain graded exercises or tests. Students could use these software packages to revise the contents learnt in the classroom, remedy the weak areas or even learn new topics prior to teachers' teaching. They could further consolidate their learning with appropriate exercises chosen for their levels of difficulty at their own pace.

(iii) a tutee - Teachers could develop their own educational programs using spreadsheets or other programming languages to suit their own teaching strategies. Students could also make use of software to explore properties of curves.

Both teachers and students of Additional Mathematics are expected to decide when to use the available technology both intelligently and critically. For example, students have to decide whether to use graphing software, or work mechanically to sketch the graph of the function $f(x) = x^3 - 3x^2 + x - 4$. Teachers have to decide whether to use computers or other devices for demonstration and which software is more appropriate for the task.

Besides, varieties of group work to facilitate collaborative learning or investigative approach in learning with IT should also be considered. Classwork or homework should emphasize concept development and understanding instead of manipulating complicated expressions or symbols or just rote memorization of formulae.

<u>Appropriate Use of Multifarious Teaching Resources</u>

Besides IT, there are other teaching resources that teachers could make use of in planning and conducting the learning and teaching activities:

- Textbooks or teaching packages
- Reference books
- Audio-visual tapes
- Instruments and other equipment for drawing shapes and making models
- Materials found around in newspapers, advertisement leaflets, maps, etc.
- Resources found in libraries / resource centres etc.

Textbooks are one of the key resources for learning and teaching. They should be used to guide students to acquire knowledge, skills and develop attitudes as well as to assimilate concepts and process information in the texts and graphics therein. Textbooks should not be treated as a mean of imparting factual knowledge or just providing exercises to drill students

on the manipulative skills. Some textbooks tend to provide exercises more than those required in the curriculum as far as the level of difficulty and the amount are concerned. Teachers should therefore exercise their discretion in selecting suitable parts to teach and avoid over-teaching or over-drilling. Besides textbooks, teachers could make use of references distributed to schools.

Mathematical language is progressively abstract. Different learning theories point out the importance of providing students with rich experiences in manipulating concrete objects as a foundation for the symbolic development. Teachers could make use of teaching aids such as 3-D models, blocks, graph boards, etc. to demonstrate the mathematical concepts and allow students to “play” around before asking students to “structure and apply” the concepts.

Materials around such as advertisement leaflets, graphs printed in the newspaper could supply up-to-date information that cannot be found in other sources and they could easily arouse students’ interest in learning. In addition, a large quantity of related materials for teachers’ reference can be obtained from libraries or various resource centres operated by the Education Department, such as Curriculum Resources Centre.

In 1990s, Internet becomes another popular source for sharing and retrieving information. Gathering and selecting information from these sources would be another major learning activities in the 21st century.

Finally, teachers should note that this document is only a guide rather than a rigid teaching plan that must be followed closely. They are encouraged to explore and discover their own teaching methods and schedules appropriate to the ability level of their students.